

Hong Kong Mathematics Olympiad (1989 – 90)

Heat Event (Group)

香港数学竞赛 (1989 – 90)

初赛项目 (团体)

1. If  $\frac{1}{a} + \frac{1}{b} = 5$  and  $\frac{1}{a^2} + \frac{1}{b^2} = 13$ , find the value of  $\frac{1}{a^5} + \frac{1}{b^5}$ .

若  $\frac{1}{a} + \frac{1}{b} = 5$  及  $\frac{1}{a^2} + \frac{1}{b^2} = 13$ , 求  $\frac{1}{a^5} + \frac{1}{b^5}$  的值。

2. There are  $N$  pupils in a class.

When they are divided into groups of 4, 1 pupil is left behind.

When they are divided into groups of 5, 3 pupils are left behind.

When they are divided into groups of 7, 3 pupils are left behind.

Find the least value of  $N$ .

某班有学生  $N$  人。

若将学生分为每 4 人一组, 有 1 人余下,

若将学生分为每 5 人一组, 有 3 人余下,

若将学生分为每 7 人一组, 有 3 人余下。

求  $N$  的最小值。

3. The coordinates of  $A$ ,  $B$ ,  $C$  and  $D$  are  $(10, 1)$ ,  $(1, 7)$ ,  $(-2, 1)$  and  $(1, 3)$  respectively.  $AB$  and  $CD$  meet at  $P$ . Find the value of  $\frac{AP}{PB}$ .

$A$ 、 $B$ 、 $C$  及  $D$  的坐标依次是  $(10, 1)$ 、 $(1, 7)$ 、 $(-2, 1)$  及  $(1, 3)$ 。 $AB$  与  $CD$  相交于  $P$ 。求  $\frac{AP}{PB}$  的值。

4. Find the remainder when  $2^{1989} + 1$  is divided by 3.

求  $2^{1989} + 1$  被 3 除所得的余数。

5. Euler was born and died between 1700 A.D. and 1800 A.D. He was  $n + 9$  years old in  $n^3$  A.D. and died at the age of 76. Find the year in which Euler died.

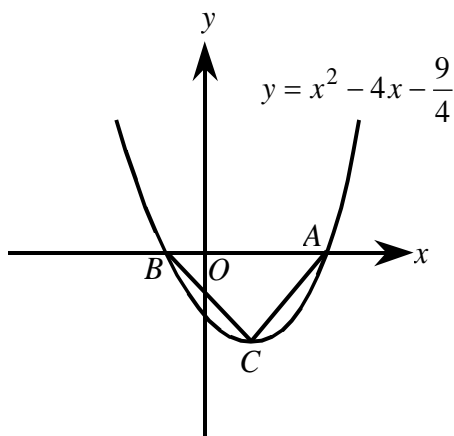
欧拉在 1700 A.D. 和 1800 A.D. 之间出生和去世。在  $n^3$  A.D. 时, 他刚好  $n + 9$  岁, 而他在 76 岁时去世。求欧拉去世的年份。

6. Let  $N!$  denote the product of the first  $N$  natural numbers, i.e.  $N! = 1 \times 2 \times 3 \times \cdots \times N$ . If  $k$  is a positive integer such that  $30! = 2^k \times \text{an odd integer}$ , find  $k$ .

设  $N!$  为首  $N$  个自然数的积, 即  $N! = 1 \times 2 \times 3 \times \cdots \times N$ 。若  $k$  是正整数使得  $30! = 2^k \times \text{一奇数}$ , 求  $k$ 。

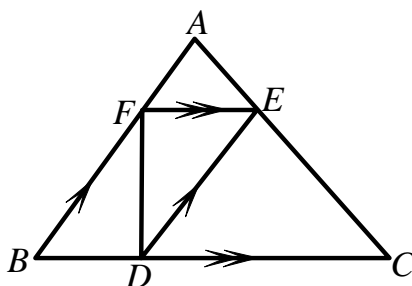
7. The graph of the parabola  $y = x^2 - 4x - \frac{9}{4}$  cuts the  $x$ -axis at  $A$  and  $B$  (Figure 1). If  $C$  is the vertex of the parabola, find the area of  $\triangle ABC$ .

抛物线  $y = x^2 - 4x - \frac{9}{4}$  的图像交  $x$ -轴于  $A$  及  $B$  (图 1)。若  $C$  是抛物线的顶点，求  $\triangle ABC$  的面积。



(Figure 1)(图 1)

8. In Figure 2,  $FE \parallel BC$  and  $ED \parallel AB$ . If  $AF : FB = 1 : 4$ , find the ratio of area of  $\triangle EDC$  : area of  $\triangle DEF$ .  
在图 2 中， $FE \parallel BC$  及  $ED \parallel AB$ 。若  $AF : FB = 1 : 4$ ，求  $\triangle EDC$  的面积 :  $\triangle DEF$  的面积。



(Figure 2)(图 2)

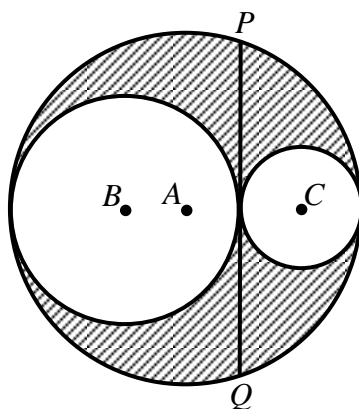
9. In the attached multiplication (Figure 3), the letters O, L, Y, M, P, I, A and D represent different integers ranging from 1 to 9. Find the integer represented by A.  
在所附乘法算式中 (图 3)，字母 O、L、Y、M、P、I、A 及 D 代表由 1 至 9 的不同整数，求 A 所代表的整数。

$$\begin{array}{r}
 \text{O L Y M P I A D} \\
 \times \quad \quad \quad \text{D} \\
 \hline
 \text{O O O O O O O O}
 \end{array}$$

(Figure 3)(图 3)

10. Three circles, with centres  $A$ ,  $B$  and  $C$  respectively, touch one another as shown in figure 4. If  $A$ ,  $B$  and  $C$  are collinear and  $PQ$  is a common tangent to the two smaller circles, where  $PQ = 4$ , find the area of the shaded part in terms of  $\pi$ .

以  $A$ 、 $B$  及  $C$  为圆心的三个圆两两相切如图 4。若  $A$ 、 $B$  及  $C$  共线， $PQ$  是两个较小圆的公切线，且  $PQ = 4$ ，试以  $\pi$  表阴影面积。



(Figure 4)(图 4)